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Online Supplement to
“The Flexibility of Models of Recognition Memory:
The Case of Confidence Ratings”

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Tables 1 and 2 give the NML penalties for seven-point and eight-point rating scales, respectively. Table 3 shows the interpolation function parameters for seven-point scales along with fit (based on the values for $n = 30$, $n = 100$, $n = 300$, $n = 1000$, and $n = 3000$) and cross-validation results (based on the values for $n = 50$ and $n = 500$) as well as selected FIA_f values. Tables 4, 5, 6, and 7 present model-recovery results for the same selection problems as in the body of the paper (Tables 4 to 7) based on data generated for seven-point scales.

Table 8 presents NML penalties for six, seven, and eight point scales and the models from the 2HTM family with different response mappings allowed for new and old items. Table 9 shows the interpolation function with fit and cross-validation results for these models and six-point and seven-point scales.

Table 1

NML Penalties (NML_P) for a Seven-Point Rating Scale

Model	p	Number of Trials with Old (and New) Items						
		30	50	100	300	500	1000	3000
1HTM _{wo}	7	11.26	12.81	15.04	18.61	20.29	22.64	26.41
1HTM _{r2}	8	11.86	13.59	16.02	20.01	21.91	24.56	28.85
1HTM _r	9	12.28	14.13	16.73	21.12	23.18	26.12	30.87
1HTM	9	13.32	15.29	18.01	22.55	24.73	27.69	32.47
2HTM _{D_o=D_n,wo}	7	10.96	12.47	14.68	18.23	19.92	22.27	26.03
2HTM _{D_o=D_n,r2}	8	11.25	12.89	15.26	19.14	21.04	23.61	27.87
2HTM _{D_o=D_n,r}	9	11.39	13.12	15.60	19.79	21.87	24.66	29.30
2HTM _{D_o=D_n}	9	12.15	14.03	16.67	21.07	23.24	26.11	30.93
EVSdT	7	12.26	13.88	16.12	19.76	21.47	23.86	27.64
UVSdT _{σ≥1}	8	13.64	15.47	18.02	22.13	24.12	26.82	31.15
UVSdT	8	14.17	16.03	18.62	22.77	24.76	27.48	31.75
2HTM _{D_o≥D_n,wo}	8	11.59	13.25	15.64	19.54	21.42	24.05	28.26
2HTM _{D_o≥D_n,r2}	9	12.17	14.00	16.57	20.89	23.00	25.89	30.59
2HTM _{D_o≥D_n,r}	10	12.53	14.47	17.19	21.83	24.09	27.21	32.38
2HTM _{D_o≥D_n}	10	13.56	15.62	18.46	23.26	25.60	28.78	34.01
2HTM _{wo}	8	11.98	13.71	16.11	20.08	21.98	24.65	28.92
2HTM _{r2}	9	12.64	14.50	17.12	21.49	23.61	26.51	31.23
2HTM _r	10	13.03	15.00	17.77	22.45	24.72	27.86	33.02
2HTM	10	14.10	16.18	19.05	23.89	26.23	29.43	34.65
DPSdT _{wo}	8	12.46	14.17	16.52	20.43	22.30	24.87	29.07
DPSdT _{r2}	9	12.68	14.48	16.92	21.16	23.19	26.03	30.71
DPSdT _r	10	12.87	14.74	17.33	21.85	24.02	27.13	32.20
DPSdT	10	13.60	15.59	18.40	23.13	25.42	28.61	33.78
MSdT ₀	8	12.75	14.56	16.96	21.00	22.95	25.61	29.89
MSdT	9	12.81	14.65	17.13	21.35	23.39	26.25	30.88

Note. p = number of parameters.

Table 2

NML Penalties (NML_P) for an Eight-Point Rating Scale

Model	p	Number of Trials with Old (and New) Items					
		30	50	100	300	500	1000
1HTM _{wo}	8	12.47	14.26	16.73	20.85	22.80	25.46
1HTM _{r2}	9	13.08	14.99	17.69	22.21	24.33	27.31
1HTM _r	11	13.76	15.89	18.95	24.12	26.65	30.13
1HTM	11	15.45	17.78	21.12	26.61	29.26	32.89
2HTM _{D_o=D_n,wo}	8	12.14	13.93	16.40	20.46	22.42	25.07
2HTM _{D_o=D_n,r2}	9	12.41	14.32	16.96	21.35	23.51	26.41
2HTM _{D_o=D_n,r}	11	12.62	14.66	17.49	22.41	24.82	28.17
2HTM _{D_o=D_n}	11	13.87	16.12	19.28	24.57	27.18	30.71
EVSdT	8	13.62	15.46	17.98	22.13	24.13	26.81
UVSdT _{σ≥1}	9	15.05	17.13	19.97	24.63	26.84	29.89
UVSdT	9	15.61	17.70	20.57	25.27	27.47	30.52
2HTM _{D_o≥D_n,wo}	9	12.80	14.70	17.33	21.76	23.89	26.83
2HTM _{D_o≥D_n,r2}	10	13.37	15.39	18.26	23.10	25.40	28.64
2HTM _{D_o≥D_n,r}	12	13.97	16.17	19.33	24.71	27.34	31.02
2HTM _{D_o≥D_n}	12	15.63	18.03	21.45	27.16	29.91	33.75
2HTM _{wo}	9	13.20	15.12	17.79	22.31	24.46	27.44
2HTM _{r2}	10	13.84	15.91	18.80	23.69	26.04	29.31
2HTM _r	12	14.48	16.76	19.90	25.33	27.98	31.68
2HTM	12	16.19	18.58	22.05	27.78	30.57	34.40
DPSdT _{wo}	9	13.83	15.73	18.36	22.77	24.89	27.78
DPSdT _{r2}	10	14.02	16.00	18.73	23.46	25.76	28.92
DPSdT _r	12	14.31	16.43	19.43	24.68	27.25	30.89
DPSdT	12	15.61	17.99	21.36	27.02	29.77	33.59
MSdT ₀	9	14.17	16.17	18.90	23.44	25.66	28.62
MSdT	10	14.24	16.28	19.09	23.83	26.16	29.32

Note. p = number of parameters.

Table 3

Interpolation Function Details and Selected FIA_f Penalties for Seven-Point Scales

Model	<i>RMSE</i>		<i>Parameters</i>			FIA_f
	Fit	CV	a	b	c	
1HTM _{wo}	0.01	0.02	4.72	0.41	2.21	2.29
1HTM _{r2}	0.02	0.02	7.24	0.43	1.16	1.19
1HTM _r	0.01	0.03	9.89	0.40	-0.43	-0.31
1HTM	0.00	0.02	7.47	0.41	1.32	1.48
2HTM _{D_o=D_n,wo}	0.01	0.03	5.78	0.46	1.87	1.85
2HTM _{D_o=D_n,r2}	0.02	0.00	8.59	0.41	0.06	0.13
2HTM _{D_o=D_n,r}	0.01	0.03	11.91	0.36	-2.24	-2.06
2HTM _{D_o=D_n}	0.02	0.03	9.91	0.43	-0.29	-0.26
EVSDT	0.00	0.02	4.10	0.47	3.53	
UVSDT _{σ≥1}	0.02	0.01	5.39	0.48	3.58	
UVSDT	0.01	0.00	3.72	0.39	4.14	
2HTM _{D_o≥D_n,wo}	0.01	0.02	7.88	0.39	0.45	0.56
2HTM _{D_o≥D_n,r2}	0.01	0.02	10.28	0.38	-0.76	-0.64
2HTM _{D_o≥D_n,r}	0.02	0.03	13.33	0.36	-2.72	-2.51
2HTM _{D_o≥D_n}	0.01	0.04	11.13	0.37	-0.89	-0.73
2HTM _{wo}	0.01	0.03	7.63	0.44	1.24	1.26
2HTM _{r2}	0.01	0.01	9.33	0.38	-0.11	0.05
2HTM _r	0.01	0.02	12.31	0.34	-2.09	-1.82
2HTM	0.01	0.03	10.40	0.36	-0.25	-0.04
DPSDT _{wo}	0.01	0.02	7.27	0.34	1.12	
DPSDT _{r2}	0.02	0.05	11.60	0.37	-0.79	
DPSDT _r	0.01	0.05	14.76	0.34	-3.12	
DPSDT	0.01	0.02	11.84	0.35	-1.23	
MSDT ₀	0.00	0.05	6.80	0.45	2.25	
MSDT	0.01	0.05	10.39	0.33	-0.77	

Note. CV = cross validation. The interpolation function for the NML penalty term is $NML_P = \frac{p}{2} \log \frac{2n}{2\pi} + an^{-b} + c$, where n is the number of signal trials and the number of noise trials so that the total N is $2n$, and p is the number of model parameters.

Table 4
Model Recovery Contrasting Pairs of Models, Recovery Rates per Model (1 vs. 2), and Overall Recovery in Percent for a 7-Point Scale

<i>n</i>	Index	2HTM _{<i>D</i>_o=<i>D</i>_n} vs. EVSDT			2HTM vs. UVSDT			UVSDT _{$\sigma \geq 1$} vs. DPSDT			DPSDT vs. MSDT			UVSDT vs. MSDT		
		1	2	Tot.	1	2	Tot.	1	2	Tot.	1	2	Tot.	1	2	Tot.
		— without response mapping (wo) ^a —											— $\sigma \geq 1^b$ —			
60	AIC	49	78	63	29	95	62	73	42	58	95	6	51	97	5	51
	BIC	49	78	63	29	95	62	73	42	58	99	4	51	99	2	51
	NML	85	51	68	68	77	72	42	83	63	92	8	50	39	82	61
	Opt.	83	53	68	77	70	74	40	87	63	98	4	51	47	76	61
			— with response mapping r^a —													
	AIC	37	88	62	38	91	65	88	8	48	49	35	42	97	5	51
	BIC	27	94	60	27	95	61	93	4	48	49	35	42	99	2	51
	NML	87	48	67	75	70	72	40	82	61	92	6	49	64	51	58
	Opt.	83	52	67	78	66	72	36	87	62	99	2	51	54	62	58
			— with response mapping r^a —													
	AIC	38	88	63	40	90	65	85	8	46	8	62	35			
	BIC	22	95	58	24	96	60	92	3	47	5	69	37			
NML	88	46	67	78	65	72	41	79	60	17	53	35				
Opt.	87	47	67	79	64	72	35	86	61	99	2	51				
		— without response mapping (wo) ^a —											— $\sigma \geq 1^b$ —			
300	AIC	64	82	73	52	96	74	72	51	62	93	17	55	92	15	54
	BIC	64	82	73	52	96	74	72	51	62	98	12	55	98	9	53
	NML	88	66	77	73	89	81	42	89	65	92	18	55	34	89	62
	Opt.	88	66	77	79	84	81	45	86	65	97	14	55	40	84	62
			— with response mapping r^a —													
	AIC	62	87	74	66	91	79	86	11	48	50	46	48	94	15	54
	BIC	54	91	73	57	94	75	92	5	48	50	46	48	98	9	54
	NML	90	61	76	81	82	81	39	88	64	90	17	54	68	59	64
	Opt.	89	63	76	80	83	82	39	88	64	97	12	54	71	56	64
			— with response mapping r^a —													
	AIC	67	84	75	73	87	80	81	11	46	9	66	38			
	BIC	56	90	73	59	92	75	88	5	47	4	74	39			
NML	76	73	75	86	77	81	41	81	61	14	63	38				
Opt.	72	80	76	86	77	81	36	89	62	96	12	54				

Note. Tot. = overall recovery rate. Opt. = optimal recovery rate.

^aRegards the 2HTM variants and the DPSDT. ^bRegards the UVSDT model.

Table 5

Model Recovery Contrasting Sets of Four Models, Recovery Rates per Model (1 to 4), and Overall Recovery in Percent for a 7-Point Scale

n	Index	2HTM $_{D_o=D_n}$, EVSDT, 2HTM, UVSDT					2HTM $_{D_o \geq D_n}$, UVSDT $_{\sigma \geq 1}$, DPSDT, MSDT				
		1	2	3	4	Tot.	1	2	3	4	Tot.
— without response mapping (wo) ^a —											
60	AIC	27	42	17	75	40	18	85	11	18	33
	BIC	35	55	12	65	42	18	85	11	18	33
	NML	59	32	28	60	45	65	58	26	17	42
— with response mapping r ^{2a} —											
60	AIC	20	45	29	72	42	19	84	1	22	32
	BIC	18	65	18	65	42	13	87	0	24	31
	NML	59	28	43	57	47	69	57	20	13	40
— with response mapping r ^a —											
60	AIC	18	45	34	72	42	21	83	1	22	32
	BIC	12	66	18	66	41	12	87	0	24	31
	NML	58	26	52	54	47	69	55	4	23	38
— without response mapping (wo) ^a —											
300	AIC	37	46	40	84	52	34	84	19	28	41
	BIC	49	61	32	74	54	34	84	19	28	41
	NML	63	48	42	72	56	68	63	39	26	49
— with response mapping r ^{2a} —											
300	AIC	38	48	58	80	56	40	82	3	30	39
	BIC	43	67	46	72	57	33	85	0	33	38
	NML	67	44	60	67	59	72	62	9	34	44
— with response mapping r ^a —											
300	AIC	41	46	68	76	58	44	79	2	29	38
	BIC	42	66	52	71	58	34	83	1	32	37
	NML	59	49	71	63	61	60	58	3	50	43

Note. Tot. = overall recovery rate.

^aRegards the 2HTM variants and the DPSDT.

Table 6

Model Recovery Contrasting A Set of Ten Models, Recovery Rates per Model (1 to 10), and Overall Recovery in Percent for a 7-Point Scale

1HTM, 2HTM $_{D_o=D_n}$, 2HTM $_{D_o \geq D_n}$, 2HTM, EVSDT, UVSDT $_{\sigma \geq 1}$, UVSDT, DPSDT, MSDT $_0$, MSDT												
n	Index	1	2	3	4	5	6	7	8	9	10	Tot.
— without response mapping (wo) ^a —												
	AIC	26	21	2	6	37	20	51	2	10	0	17
	BIC	35	27	1	4	48	17	44	1	7	0	18
	NML	25	53	6	8	26	34	22	4	13	1	19
— with response mapping r ^{2a} —												
	AIC	23	19	2	10	42	20	49	0	10	0	18
	BIC	22	17	0	5	62	18	44	0	9	0	18
	NML	38	52	7	11	20	34	20	2	8	1	19
— with response mapping r ^a —												
	AIC	27	16	1	10	43	20	48	0	14	0	18
	BIC	21	10	0	4	64	18	44	0	12	0	17
	NML	43	52	6	12	19	34	17	1	7	1	19
— without response mapping (wo) ^a —												
	AIC	34	32	6	19	41	19	56	10	20	2	24
	BIC	45	41	4	13	57	17	50	5	16	1	25
	NML	35	58	13	13	41	31	30	11	21	3	26
— with response mapping r ^{2a} —												
	AIC	33	36	8	25	44	19	54	1	19	2	24
	BIC	36	41	4	17	65	18	49	0	17	1	25
	NML	45	63	16	17	36	30	28	2	16	3	26
— with response mapping r ^a —												
	AIC	41	38	7	25	43	19	51	0	21	2	25
	BIC	40	39	3	17	65	18	48	0	21	1	25
	NML	43	56	15	19	42	30	25	1	19	4	25

Note. Tot. = overall recovery rate.

^aRegards the models 1HTM, the 2HTM variants, and DPSDT.

Table 7

Model Recovery Excluding Non-Diagnostic Data, Recovery Rates per Model (1 to 7), and Overall Recovery in Percent for a 7-Point Scale

		2HTM _{D_o ≥ D_n} , 2HTM UVSDT _{σ ≥ 1} , UVSDT, DPSDT, MSDT ₀ , MSDT							
<i>n</i>	Index	1	2	3	4	5	6	7	Tot.
		— without response mapping (wo) ^a —							
60	AIC	7	17	31	64	10	24	0	22
	BIC	7	17	31	64	11	24	0	22
	NML	18	19	60	32	11	29	2	24
			— with response mapping r ^{2a} —						
	AIC	8	23	32	65	1	19	0	21
	BIC	5	16	32	66	0	22	0	20
60	NML	22	27	61	30	5	20	2	24
			— with response mapping r ^a —						
	AIC	6	25	33	64	0	20	1	21
	BIC	3	15	34	66	0	22	0	20
	NML	22	31	64	27	2	19	2	24
			— without response mapping (wo) ^a —						
300	AIC	16	34	29	64	24	36	3	29
	BIC	16	34	29	64	24	37	1	29
	NML	33	26	51	38	26	40	5	31
			— with response mapping r ^{2a} —						
	AIC	21	44	29	63	4	30	4	28
	BIC	16	38	31	65	1	35	2	27
300	NML	44	33	50	37	7	34	5	30
			— with response mapping r ^a —						
	AIC	17	44	29	59	1	36	5	27
	BIC	13	36	31	63	0	41	2	27
	NML	37	34	51	33	2	38	8	29

Note. Tot. = overall recovery rate.

^aRegards the 2HTM variants and the DPSDT.

Table 8

NML Penalties (NML_P) for Models with Different Response Mappings for Old and New Items

Model	p	Number of Trials with Old Items (Same as for New Items)								
		30	50	100	300	500	1000	3000	5000	10000
— Six-Point Scale ^a —										
2HTM _{$D_o=D_n, r_2$}	8	10.36	11.87	14.11	17.86	19.72	22.24	26.42	28.39	31.11
2HTM _{$D_o=D_n, r$}	10	10.81	12.54	15.04	19.41	21.54	24.60	29.63	32.05	35.37
2HTM _{$D_o=D_n$}	10	12.53	14.51	17.38	22.16	24.43	27.66	32.90	35.37	38.72
2HTM _{$D_o \geq D_n, r_2$}	9	11.11	12.75	15.14	19.24	21.22	24.00	28.56	30.77	33.74
2HTM _{r_2}	9	11.52	13.22	15.65	19.78	21.80	24.60	29.20	31.40	34.41
— Seven-Point Scale —										
2HTM _{$D_o=D_n, r_2$}	9	11.61	13.36	15.86	20.11	22.17	25.05	29.73		
2HTM _{$D_o=D_n, r$}	11	12.02	13.93	16.74	21.57	23.95	27.30	32.83		
2HTM _{$D_o=D_n$}	11	13.67	15.85	18.95	24.23	26.78	30.30	36.08		
2HTM _{$D_o \geq D_n, r_2$}	10	12.40	14.25	16.89	21.53	23.72	26.88	32.01		
2HTM _{$D_o \geq D_n, r$}	12	12.81	14.81	17.75	22.77	25.26	28.82	34.65		
2HTM _{$D_o \geq D_n$}	12	14.21	16.46	19.64	25.16	27.84	31.57	37.69		
2HTM _{r_2}	10	12.82	14.70	17.43	22.07	24.30	27.50	32.65		
2HTM _{r}	12	13.26	15.27	18.23	23.31	25.81	29.37	35.26		
2HTM	12	14.57	16.81	20.08	25.65	28.35	32.10	38.28		
— Eight-Point Scale —										
2HTM _{$D_o=D_n, r_2$}	10	12.79	14.70	17.52	22.26	24.58	27.80			
2HTM _{$D_o=D_n, r$}	14	13.44	15.64	18.93	24.70	27.60	31.66			
2HTM _{$D_o=D_n$}	14	16.10	18.75	22.62	29.18	32.37	36.85			
2HTM _{$D_o \geq D_n, r_2$}	11	13.57	15.61	18.60	23.69	26.15	29.64			
2HTM _{$D_o \geq D_n, r$}	15	14.28	16.55	19.94	25.83	28.76	32.90			
2HTM _{$D_o \geq D_n$}	15	16.49	19.16	23.05	29.64	32.87	37.40			
2HTM _{r_2}	11	13.99	16.07	19.11	24.27	26.79	30.25			
2HTM _{r}	15	14.71	17.01	20.44	26.37	29.27	33.44			
2HTM	15	16.78	19.46	23.36	29.98	33.21	37.73			

Note. p = number of parameters.

^aNot all 2HTM models are identified for six-point scales.

Table 9

Interpolation Function Details and FIA_f Penalties for the Models from Table 8 and Six-Point and Seven-Point Scales

Model	<i>RMSE</i>		<i>Parameters</i>			FIA_f
	Fit	CV	<i>a</i>	<i>b</i>	<i>c</i>	
— Six-Point Scale —						
$2HTM_{D_o=D_n, r2}$	0.02	0.01	11.66	0.42	-1.41	-1.35
$2HTM_{D_o=D_n, r}$	0.03	0.03	17.86	0.37	-5.61	-5.37
$2HTM_{D_o=D_n}$	0.01	0.03	12.05	0.39	-1.95	-1.79
$2HTM_{D_o \geq D_n, r2}$	0.02	0.01	13.47	0.35	-3.11	-2.85
$2HTM_{r2}$	0.02	0.01	12.80	0.36	-2.36	-2.16
— Seven-Point Scale —						
$2HTM_{D_o=D_n, r2}$	0.00	0.01	11.92	0.39	-1.66	-1.50
$2HTM_{D_o=D_n, r}$	0.01	0.01	18.16	0.34	-6.13	-5.63
$2HTM_{D_o=D_n}$	0.01	0.02	13.97	0.41	-2.19	-2.05
$2HTM_{D_o \geq D_n, r2}$	0.01	0.03	14.92	0.38	-3.04	-2.87
$2HTM_{D_o \geq D_n, r}$	0.02	0.03	21.07	0.29	-8.63	-7.76
$2HTM_{D_o \geq D_n}$	0.01	0.03	17.14	0.35	-4.51	-4.18
$2HTM_{r2}$	0.00	0.03	14.14	0.38	-2.33	-2.18
$2HTM_r$	0.02	0.03	20.87	0.30	-7.85	-7.07
$2HTM$	0.01	0.00	16.62	0.36	-3.81	-3.49

Note. CV = cross validation. The interpolation function for the NML penalty term is $NML_P = \frac{p}{2} \log \frac{2n}{2\pi} + an^{-b} + c$, where n is the number of signal trials and the number of noise trials so that the total N is $2n$, and p is the number of model parameters.